The reactions $pp \to pp\pi^0$ and $pp \to d\pi^+$ at threshold: The role of the isoscalar πN scattering amplitude

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Abstract

We examine the role of the elementary isoscalar pion–nucleon scattering amplitude in the description of the processes $pp \to pp\pi^0$ and $pp \to d\pi^+$ at threshold. We argue that the presently used tree level dimension two approximation used in chiral perturbation theory is insufficient as input by direct comparison with the πN scattering data. We also show that a successful semi–phenomenological boson–exchange model does better in the description of these data. The influence of the violation of crossing symmetry in the meson–exchange model has to be studied in more detail. We stress that further investigations of the process $pp \to d\pi^+$ can pave the way to a deeper understanding of the pion dominated part of the transition operator.

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1. The high precision data for the processes $pp \to pp\pi^0$ and $pp \to d\pi^+$ in the threshold region [1][2] have spurred a flurry of theoretical investigations. The first data on neutral pion production were a big surprise because the experimental cross sections turned out to be a factor of five larger than the theoretical predictions. These included the direct (Born) graph (fig.1a) and the on-shell rescattering (fig.1b), with the pion-nucleon (πN) T-matrix replaced by the scattering length. Subsequently, it was argued that heavymeson exchanges (fig.1c) might be able to remove this discrepancy [3]. On the other hand it was shown [4][5] that the off-shell behaviour of the full πN T-matrix also enhances the cross section considerably. In [5], the pion-nucleon T-matrix was calculated within a semi-phenomenological meson-exchange model, based on an effective Lagrangian fullfilling certain requirements from chiral symmetry (as discussed below). Recently this result has been questioned based on calculations within the framework of tree-level chiral perturbation theory (CHPT) including dimension two operators [6, 7, 8, 9]. In these papers, CHPT has been used to constrain the long-range pion-exchange contributions. In fact, the chiral perturbation theory approach, which abides to all symmetry requirements including crossing^{#6}, is believed to lead to a deeper understanding of the success of the meson-exchange picture, as first stressed by Weinberg [10]. It is therefore striking that the calculations for $pp \to pp\pi^0$ performed so far lead to a marked difference in the role of the so-called rescattering contribution, which interferes constructively with the direct production in the Jülich model and destructively in the chiral framework, respectively. Note that while there is still debate about the actual numerical treatment (co-ordinate versus momentum space) and the ensuing size of the rescattering contribution in the chiral perturbation theory approaches [9], the sign difference to the meson–exchange model can be considered a genuine feature. It is exactly this point which we wish to address in this letter. We argue that the treatment underlying the isoscalar pion–nucleon scattering amplitude and the related transition operator for the process $NN \to NN\pi$ in the chiral framework is not yet sufficiently accurate and thus the resulting rescattering contribution should be considered an artifact of this approximation. Clearly, this does not mean that chiral perturbation theory is invalid but rather that higher order (one loop) effects need to be accounted for. This statement can be made very transparent if one considers in detail the isoscalar πN scattering amplitude. We stress, however, that in the meson-exchange model crossing symmetry is violated and we can not exclude the possibility that this is in fact the reason for the constructive interference. To gain a deeper understanding of the underlying direct and rescattering contributions, we propose to study in more detail the process $pp \to d\pi^+$ since heavy meson exchanges play no role in that case and one thus has a better chance to extract the pionic (chiral) physics which underlies the parameterization of the $NN \to NN\pi$ transition operator. We furthermore mention that for our investigation mainly focusing on the strength of the S-wave rescattering, the explicit inclusion of the $\Delta(1232)$ is not needed. Clearly, for a precise comparison with the data, this approximation can not be maintained. Some of the effects of the Δ are encoded in the numerical values of the low-energy constants c_i . Further dynamical effects related to the Δ only appear at higher orders together with many other terms. A systematic inclusion of such effects goes beyond the scope of this letter. We stress that our intention

^{#6}Note, however, that at tree level unitarity is violated since all amplitudes are real.

is to critically assess the accuracy of the presently available CHPT calculations for pion production in pp collisions.

The important feature of this reaction is the large momentum mismatch between the initial and the final nucleon–nucleon state: the initial relative momentum is given by $p_{in}^2 = mM_{\pi} + M_{\pi}^2/4$, whereas close to threshold the final one is compatible with zero. This huge difference leads to momentum transfers in the order of p_{in} , at least if one restricts oneself to S–wave production. Let us concentrate first on the so–called rescattering contribution depicted in fig. 1b. To get a feeling for the relevant momentum transfer, consider the reaction at threshold and neglect the pp final–state interactions. One finds for the four–momentum of the exchanged pion, $k^2 = -mM_{\pi} \simeq -7M_{\pi}^2 \simeq -0.1\,\text{GeV}^2$, which is deeply space–like. We remark that in single pion electroproduction off protons, one–loop effects for such photon virtualities are fairly large [11] substantiating our previous statement that one has to go beyond tree level in CHPT.

2. The study of pion–nucleon scattering in chiral perturbation theory has already a long history. In the relativistic approach, Gasser et al. [12] constructed the full off-shell πN amplitude to one loop in the chiral expansion. Their work focused mainly on the determination of the pion–nucleon σ –term and the related scattering lengths. A systematic comparison with the low energy πN scattering data was, however, never presented. In the framework of heavy baryon CHPT, the chiral corrections to the S–wave scattering lengths were worked out by Bernard et al. [13]. In particular, it was shown that the isoscalar scattering length a^+ can be expressed as (to one loop)

$$4\pi \left(1 + \frac{M_{\pi}}{m}\right) a^{+} = \frac{2M_{\pi}^{2}}{F_{\pi}^{2}} \left(c_{2} + c_{3} - 2c_{1} - \frac{g_{A}^{2}}{8m}\right) + \frac{3g_{A}^{2}M_{\pi}^{3}}{64\pi F_{\pi}^{4}} + \mathcal{O}(M_{\pi}^{4}) , \qquad (1)$$

with M_{π} (m) the charged pion (nucleon) mass, $F_{\pi} = 92.4$ MeV the pion decay constant and $g_A = 1.26$ the nucleon axial-vector coupling constant. The c_i are low-energy constants (LECs) not fixed by chiral symmetry that appear in the dimension two chiral Lagrangian,

$$\mathcal{L}_{\pi N}^{(2)} = \bar{N} \left(c_1 \operatorname{Tr}(\chi_+) + (c_2 - \frac{g_A^2}{8m}) (v \cdot u)^2 + c_3 u \cdot u + \ldots \right) N , \qquad (2)$$

where the ellipsis denotes further terms not needed here, u_{μ} is the standard axial-vector, which transforms homogeneously under non-linearly realized chiral symmetry, v_{μ} the nucleons' four-velocity ($v^2 = 1$), $\chi_{+} \sim M_{\pi}^2$ parameterizes the explicit chiral symmetry breaking due to the light quark masses (and thus is directly related to the σ -term) and N is the velocity-projected nucleon isodoublet, $N = (1 + \psi)\Psi/2$ (for a detailed review, see [14]). The c_i are finite since loop corrections start to contribute only at the next order in the chiral expansion. The precise values of these LECs will be discussed below. It is important to note that there are large cancelations between the one loop contribution of order M_{π}^3 and the kinematically 1/m corrections at order M_{π}^2 . This makes the actual value of a^+ very sensitive to the values of the LECs c_i . The scattering amplitude in the tree approximation to second order in small momenta is readily derived from Eq.(2) since to leading order in pion fields, $u_{\mu} = -i\partial_{\mu}\phi/F_{\pi} + \mathcal{O}(\phi^2)$. This is the accuracy to which this amplitude has been used in chiral perturbation theory approaches to $pp \to pp\pi^0$ in the threshold region. In that case, the produced pion is almost at rest, i.e. has a very small

three–momentum (denoted q), whereas the exchanged pion typically carries a momentum k of a few hundred MeV. The explicit form of the half–off shell scattering amplitude $T^+(q,k)$ is e.g. given in [6]. Clearly, the sensitivity to the precise values of the LECs observed for the scattering length carries over to the scattering amplitude. Note that this isoscalar scattering amplitude gives rise to the so–called rescattering contribution.

To proceed, we briefly summarize what is known about the LECs c_i . First, it is important to notice that there is an ambiguity in pinning down their values. These depend on the order one is working. From a tree level fit to pion–nucleon scattering (sub)threshold parameters including the dimension two operators, one obtains [15]

$$c_1 = -0.64 \pm 0.14$$
, $c_2 = 1.78 \pm 0.10$, $c_3 = -3.90 \pm 0.09$, (3)

with all numbers given in GeV^{-1} . Note that the sum $c^+ = c_2 + c_3 - 2c_1$ which enters the isoscalar amplitude, is $c^+ = -0.84$. In this determination, the empirical value for a^+ was not used simply because it is badly determined and also, it is not possible to describe all (sub)threshold data that are sensitive to the c_i by one simultaneous fit at order M_{π}^2 (as discussed in detail in [16]). We therefore have also performed calculations with $c^+ = 0.459$ and 0.005 GeV^{-1} corresponding to the the conservative band of $a^+ = \pm 10 \cdot 10^{-3}/M_{\pi}$, respectively. Since there are no compelling arguments which of the c_i should be readjusted, we have changed either one of them so as to get these values for a^+ .

To one loop order, one can determine these LECs from a set of seven observables which are given by tree graphs including the c_i and finite loop corrections but have no contribution from the 24 dimension three LECs [16]. One finds

$$c_1 = -0.93 \pm 0.10 \; , \quad c_2 = 3.34 \pm 0.20 \; , \quad c_3 = -5.29 \pm 0.25 \; .$$
 (4)

The differences between the numbers given in Eq.(3) and Eq.(4) are sizeable showing that the one–loop corrections are not small and thus have to be taken into account. We remark that in this case $c^+ = -0.09$ and $a^+ = -4.7 \cdot 10^{-3}/M_{\pi}$, well within the empirical band. Note that c^+ is an order of magnitude smaller than any of the LECs individually and is well within the uncertainty of the LECs. This observation is at the origin of the statement that to understand the empirical value of a^+ , one has to know these LECs very precisely. Mojzis [17] has recently performed a complete one–loop calculation of the πN scattering amplitude, including the dimension three counterterms. A fit to the known S-, P-, D- and F-wave threshold parameters as well as to the pion–nucleon σ –term and the Goldberger–Treiman discrepancy allows to pin down all LECs. Remarkably, the dimension two LECs of relevance here come out completely consistent with the values found in [16]. To assess the sensitivity to the LECs, we will use both sets even so we will only work to second order in the chiral expansion. This was also done in [6][8]. At this point, one might already suspect that this approximation is not sufficient.

In contrast, the isovector amplitude is dominated by the Weinberg term, i.e. the $\pi\pi\bar{N}N$ vertex which stems from the chiral covariant derivative and has it strength given entirely in terms of $1/4F_{\pi}^2$. The chiral corrections have also been calculated [13][18], but since they are small, they are not important for the following arguments. By forming appropriate linear combinations of the isoscalar and isovector amplitudes, one can construct the phase shifts S_{11} and S_{31} . They consist of a Born and a non–Born piece and take the form (expanding

the partial waves, not the invariant amplitudes, to second order in small momenta)

$$S_{11} = \sqrt{\omega^2 - M_{\pi}^2} \left(1 - \frac{\omega}{m} \right) \frac{1}{4\pi F_{\pi}^2} \left\{ -\omega + \frac{1}{2m} (M_{\pi}^2 - \omega^2) + 2M_{\pi}^2 c_1 - 2\omega^2 (c_2 + c_3) + \frac{g_A^2}{4} \left(\frac{3\omega^2}{2m} - \frac{1}{3m} \left[4M_{\pi}^2 + \frac{M_{\pi}^4}{\omega^2} - \frac{7\omega^2}{2} \right] \right) \right\} ,$$

$$S_{31} = \sqrt{\omega^2 - M_{\pi}^2} \left(1 - \frac{\omega}{m} \right) \frac{1}{4\pi F_{\pi}^2} \left\{ \frac{\omega}{2} - \frac{1}{4m} (M_{\pi}^2 - \omega^2) + 2M_{\pi}^2 c_1 - 2\omega^2 (c_2 + c_3) + \frac{g_A^2}{6m} \left[4M_{\pi}^2 + \frac{M_{\pi}^4}{\omega^2} - \frac{7\omega^2}{2} \right] \right\} , \tag{5}$$

with ω the pion cms energy. The phase shifts are shown in fig. 2 for the c_i as given in Eqs.(3,4) in comparison to the data. In both cases, the prediction deviate already at low energies from the data. Due to the approximations involved, in both cases the partial waves increase quadratically at higher energies, i.e. one has to include loop effects if one wishes to find a good representation of the πN phase shifts for energies up to 100 MeV or higher. We have also calculated these phases at tree level (dimension two) with c^+ adjusted so as to give the empirical range of a^+ as discussed above. Enhancing e.g. c_2 by a factor of 1.7 in the tree level fit leads to a good description of the phase shifts up to $T_{\rm cms} \simeq 125$ MeV. Such an enhancement is, however, incompatible with some subthreshold parameters, for detailed study see [16]. We therefore do not consider this a justified way of describing elastic pion—nucleon scattering for the energies under consideration. Furthermore, at higher energies the unphysical quadratic growth shows up again. We remark that the behavior of the phase shifts shown has direct consequences for the transition operator relevant for $NN \to NN\pi$ as discussed below.

3. There is a different way of constructing the $NN \to NN\pi$ transition operator that is by means of a meson-exchange model as e.g. the one constructed by Schütz et al. [19]. We briefly summarize its salient features without going into any kind of detail (see also the discussion in ref.[5]). The pseudo-potential contains in addition to the nucleon- and Δ -pole and exchange diagrams ρ and σ t-channel-exchanges. However, the latter two do not appear as sharp particles but are constructed from the correlated two-pion exchange by use of a dispersion integral. The potential is then iterated in a relativistic Lippmann— Schwinger equation within the framework of time-ordered perturbation theory to ensure unitarity. To get finite results, the vertices are supplemented by meson-nucleon form factors. This approach gives a unique prescription to go off shell. The parameters are then fitted to the πN phase shifts up to lab energies of about 500 MeV under the constraint that the scattering lengths come out to be in the empirical ranges. In addition it turns out that the model produces a reasonable value for the πN σ -term. As it was stated above the value of the isoscalar scattering length is compatible with zero. In the mesonexchange model, one finds $a^+ = -1.7 \cdot 10^{-3}/M_{\pi}$. This small value is a consequence of a cancelation between the σ -exchange and the iterated ρ -exchange.^{#7} However, a priori there is no reason for this cancellation to remain valid when leaving the on-shell point. We actually observe a strong momentum dependence in the half-off-shell isoscalar T-matrix at threshold. The sign of this function is fully determined by the fit to the on-shell data.

 $^{^{\#7}}$ Note that on the potential level, the ρ only contributes to the isovector channel.

The drawback of this approach is the blatant violation of crossing symmetry which is due to the iteration in the Lippmann–Schwinger equation to generate the full T–matrix. To be precise, the iteration generates contributions with powers in ν which should be zero if crossing were to be respected. Some of those coefficients turn out to have non–negligible strength. However, one can calculate the subthreshold parameters, i.e. the expansion of the invariant amplitudes around the point $\nu = t = 0$ in the $\nu - t$ plane, which are allowed by crossing. For doing that, we used for convenience model 1 of ref.[19]. These are tabulated in table 1 (for definitions, see ref. [20]).

Amp.	x_{00}	Exp.	x_{01}	Exp.	x_{02}	Exp.
A^+	-1.54	-1.46 ± 0.10	1.20	1.14 ± 0.02	0.018	0.036 ± 0.003
A^-/ν	-11.46	-8.83 ± 0.10	-0.28	-0.374 ± 0.002	-0.028	-0.015 ± 0.002
B^+/ν	-2.6	-3.54 ± 0.06	0.13	0.18 ± 0.01	-0.004	-0.01
B^-	12.64	10.36 ± 0.10	0.20	0.24 ± 0.01	-0.02	0.025 ± 0.002

Table 1: Subthreshold parameters calculated in the meson–exchange model after neglecting the crossing–violating terms (which affects in particular A^-/ν and B^+/ν). The data are from ref. [20] and the units are appropriate powers of the inverse pion mass.

4. We now turn to the reaction $pp \to pp\pi^0$. The pertinent transition matrix element is calculated in the distorted wave Born approximation, allowing us to properly include the final state nucleon–nucleon interaction. This was shown to be crucial to get the correct energy–dependence of the reaction under consideration [21].

As it was stated above, it is only the isoscalar T-matrix that enters in this reaction. It turns out, however, that the results of the two approaches described before are very different. The chiral tree calculation leads to a destructive interference with respect to the direct contribution (cf. fig. 1a) (see e.g. [6]), whereas in the meson-exchange model one finds a constructive interference [5] (the main difference of the curves presented here to the ones in [5] is that now the backward-in-time rescattering is considered in addition; details will be described elsewhere [22]). The results for the different approaches are shown in fig. 3 (using the LECs given in Eq.(3) for the chiral calculation). In the numerical treatment of the chiral tree level approach we deviate from the strict CHPT treatment of the production operator to the extent that we apply a form factor on the pion-nucleon vertex. This was done not only to achieve better convergence of the integral but also to prevent the production operator to show the abovementioned unphysical linear growth in the momentum. Although this treatment changes the quantitative results it does not effect their qualitative behavior. We have checked that while the actual strength of the rescattering contribution in CHPT is very sensitive to the values of the LECs c_i , its sign is uniquely fixed to this order (dimension two). In both results there is a remaining discrepancy to the data. A possible mechanism to fill this gap are the so called heavy meson exchanges [3]. However, their sign is such that they give a constructive interference with the direct term (actually allowing for a good description of the data [22]). Heavy meson exchange will therefore even worsen the discrepancy for the tree level chiral calculation [9].

We also applied for the first time the CHPT approach to the reaction $pp \to d\pi^+$. Here neither the direct contribution nor the heavy meson exchanges contribute considerably [23]. In this reaction a charge exchange is allowed and thus the much larger isovector channel of the πN system can contribute. To make the comparison of the different approaches in this reaction, we show in fig. 4 the effect of including the isoscalar rescattering with respect to the isovector contribution. As before, the interference pattern of the two approaches is totally different. While in case of the meson–exchange picture the isoscalar rescattering is bringing the theory in agreement with the data, using tree level chiral perturbation theory worsens the description. We conclude from this comparison that more detailed studies of this reaction might eventually lead to a deeper understanding of the relative size of the isovector and isoscalar components entering the $NN \to NN\pi$ transition operator.

5. To summarize, we have presented a detailed study about the relative sign of the rescattering and direct terms for single pion production in pp collisions in the framework of meson-exchange and tree level dimension two chiral perturbation theory calculations. While the latter is a more fundamental approach, the underlying (small) isoscalar scattering amplitude and the related $NN \to NN\pi$ transition operator are not sufficiently accurate to the order they have been treated so far for the momenta involved. We can make this statement even more precise. Only at one loop order, terms proportional to the squared momentum of the exchanged pion appear in the πN amplitude. We expect exactly these terms to control the πN T-matrix at large space-like momenta. Neglecting these terms is most probably at the heart of the sign discrepancy discussed here. We stress again that this does not mean that chiral perturbation theory can not be used, but rather that one has to work out higher order (loop) effects before one can draw decisive conclusions. In any case, a sufficiently improved CHPT calculation will have to be supplemented by heavy meson exchanges (or contact terms representing this short-range physics) to be able to describe the $pp \to pp\pi^0$ data. The more successful meson–exchange model, which describes the on-shell πN scattering data and does much better for $pp \to pp\pi^0$, has the deficiency of violating crossing symmetry. One can therefore not definitively conclude that this is the reason for the relative sign difference to the chiral approach, although this appears to be a fairly exotic possibility. In that respect, more attention should be focused on $pp \to d\pi^+$ since it is much less sensitive to the heavy meson exchanges but still sensitive enough to the isoscalar scattering amplitude.

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Figures

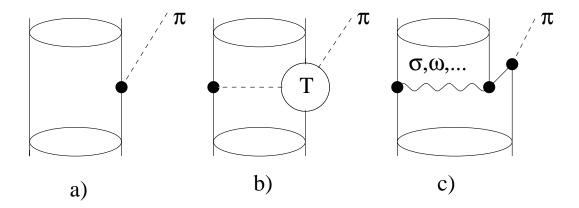


Figure 1: Different contributions considered for the meson production. a), b) and c) are referred to as the direct, rescattering and heavy meson—exchange terms, in order. NN interactions are depicted by the blobs.

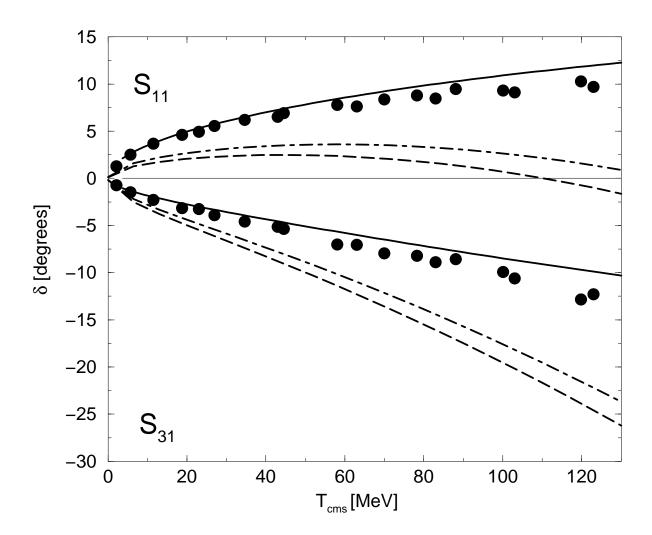


Figure 2: Comparison of the πN phase shifts S_{11} and S_{31} for the meson–exchange model (solid lines) and the tree level CHPT calculation based on parameter sets 1, Eq.(3) (dashed line) and 2, Eq.(4) (dot–dashed lines) with the data.

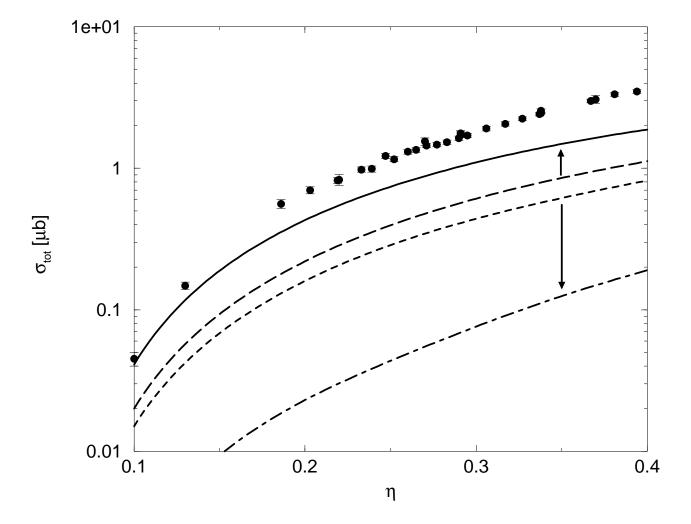


Figure 3: Results for S-wave $pp \to pp\pi^0$. The long-dashed line is the result of the rescattering contribution from the meson exchange-model. Adding the direct contribution to this gives the upper solid line (both interfere constructively as indicated by the arrow). The short-dashed line is the result of the rescattering contribution from tree level CHPT (parameters from Eq.(3)). Adding the same amplitude for the direct contribution as before leads to the lower solid line (the arrow indicates the destructive interference). Note that heavy meson exchanges are not included. η is the maximal pion momentum in units of the pion mass.

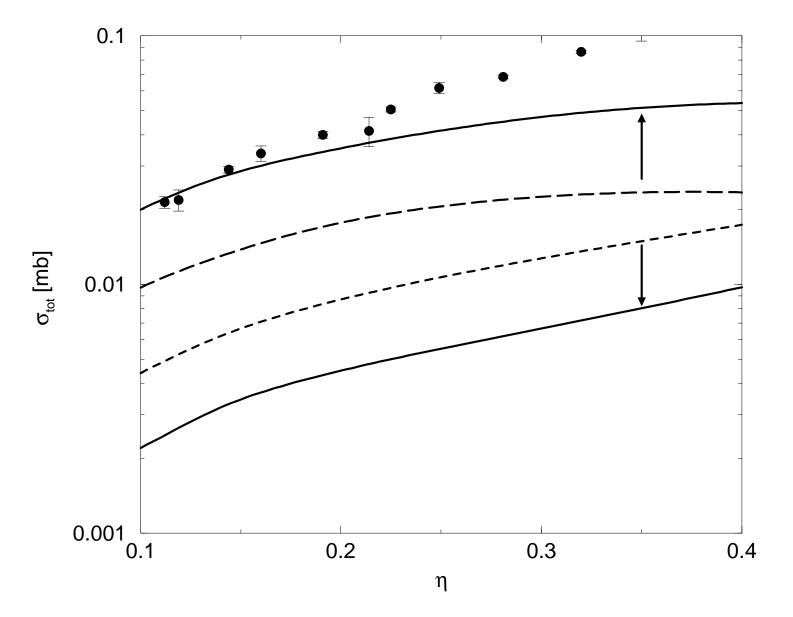


Figure 4: Influence of the isoscalar πN amplitude on S-wave $pp \to d\pi^+$. The long-dashed line shows the isovector rescattering of the meson-exchange model. Adding the isoscalar contribution leads to the upper solid line. The short-dashed line is the isovector rescattering contribution of the tree level CHPT. Adding the CHPT-isoscalar amplitude to this leads to the lower solid curve, which substantially deviates from the data. Note that the effects of heavy meson exchanges and the direct term are small and not included here. In this case, $\eta = |\vec{q}|/M_{\pi}$.